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Quantum nonlocality of Heisenberg XX model with site-dependent coupling strength

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Abstract

We show that the generalized Bell inequality is violated in the extended Heisenberg model when the temperature is below a threshold value. The threshold temperature values are obtained by constructing exact solutions of the model using the temperature-dependent correlation functions. The effect due to the presence of an external magnetic field is also illustrated.

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1. Introduction

An intriguing aspect of quantum mechanics is the lack of a local realistic description that could reproduce the necessary correlations for the experimental outcomes in composite systems [1]. This lack of local realism can be investigated using the entangled state as discussed in the original seminal paper by Einstein *et al*. Nowadays, we recognize the importance of entanglement as a valuable resource for quantum information processing and communication. Its usefulness has since been demonstrated clearly in processes such as quantum teleportation [2, 3], quantum computation [4] and quantum cryptography [5].

However, concepts such as entanglement and its implications concerning the non-existence of a local realism in quantum mechanics have a more fundamental role in quantum mechanics. The issue of 'locality' as well as the notion of quantum measurements has given rise to some of the recent and modern interpretations of quantum mechanics as well as a better understanding of quantum phenomena [6]. It is also amidst all these theoretical constructs that Bell proposed an inequality that could rule out the hidden variable description of quantum mechanics [7]. Since then, several variants of Bell inequalities, some of which were more amenable for experimental investigations, have been derived for two-body correlation functions to investigate the existence of local realism [8].

Recently, there has been much work on the implementation of quantum processing on solid state devices. In this paper, we study the thermal states in a system of interaction spins and investigate its quantum 'nonlocality'. An interesting type of entanglement, thermal entanglement, was studied in the context of the Heisenberg XXX [9, 10], XX [11] and XXZ [12] models. The Heisenberg model has been shown to be a potential candidate as a model for spin–spin interaction in a solid state quantum computer [13]. Being the large Coulomb repulsion limit of the Hubbard model, it has been partially realized in quantum dots [13], nuclear spins [14] and optical lattices [15]. In a recent work, Imamoglu *et al* [16] have realized quantum information processing using quantum dot spins and cavity QED, and obtained an effective interaction Hamiltonian based on the XY spin chain between two quantum dots. The effective Hamiltonian was shown to be capable of constructing the controlled-not gate [16]. The XY Hamiltonian is given by

$$H = \sum_{n=1}^{N} \left(J_1 S_n^x S_{n+1}^x + J_2 S_n^y S_{n+1}^y \right) \tag{1}$$

where $S^i = \sigma^i/2\,(i = x, y, z)$ and σ^i are Pauli operators. When $J_1 = J_2$, the XY model becomes the XX model. In the XY model, the interaction strength between neighbouring sites is usually assumed to be independent of the sites. In most solid state models however, the inter-site coupling strength is site dependent. In this paper we consider an extended quantum XX model in which the interaction strength assumes a particular site-dependent form.

This paper is organized as follows. In section 2, solutions of the extended XX model for four particles are given. In section 3, we construct the temperature-dependent correlation functions in terms of the thermal equilibrium state and investigate the violation of the Bell inequality for the thermal state. The threshold temperature is given. We also point out that the eigenstates of the extended XX model do not realize maximal violation of the Bell inequality. The effect of an external magnetic field is discussed in section 5 and we end with some discussions in the final section.

2. Solution of the extended XX model

The extended XX Heisenberg model is described by the Hamiltonian

$$\begin{aligned} H &= 2\sum_{n=1}^{N-1} J_{n,n+1}\left(\sigma_n^x \sigma_{n+1}^x + \sigma_n^y \sigma_{n+1}^y\right) \\ &= \sum_{n=1}^{N-1} J_{n,n+1}\left(\sigma_n^+ \sigma_{n+1}^- + \sigma_n^- \sigma_{n+1}^+\right) \end{aligned} \tag{2}$$

where $J_{n,n+1} = \sqrt{n(N-n)}$ is the coupling strength between lattices n and $n+1$. Obviously, the Hamiltonian H describes a nearest-neighbour interaction spin chain. Interestingly, such a Hamiltonian has been shown to be useful for perfect state transfer in quantum spin networks [17]. The Hamiltonian H possesses 2^N complete and orthonormal eigenstates.

When spin chains are subjected to environmental disturbance, they inevitably become thermal equilibrium states. The state of a system at finite temperature T is given by the Gibb's density operator $\rho(T) = \exp(-H/kT)/Z$, where $Z = \mathrm{Tr}[\exp(-H/kT)]$ is the partition function, H is the system Hamiltonian and k is the Boltzmann constant, which is set to unity for convenience in this paper. At high temperature, the thermal state becomes maximally mixed and does not violate Bell inequalities of any kind. It is therefore interesting to consider the critical temperature at which a Bell inequality will be violated. For a two-qubit system, we have the original Bell inequality. For an arbitrary number of qubits, we have the Zukowski–Brukner inequality [8].

Unfortunately it is not possible to test the Zukowski–Brukner inequality for three qubits in this case since the correlation functions defined below are zero. Therefore, in this paper, we first focus on the next non-trivial case of a 4-qubit system and test the violation of the local realistic description using the Zukowski–Brukner inequality. The extension to an arbitrary number of sites, albeit complicated, can also be done in the same manner. The Hamiltonian has 16 eigenvalues

$$\begin{aligned}
&E_0 = E_7 = E_8 = E_{15} = 0, \\
&E_3 = E_{13} = -1, \qquad E_4 = E_{14} = 1, \\
&E_6 = -2, \qquad E_9 = 2, \\
&E_1 = E_{11} = -3, \qquad E_2 = E_{12} = 3, \\
&E_5 = -4, \qquad E_{10} = 4.
\end{aligned} \tag{3}$$

The corresponding eigenstates $\{|\phi_0\rangle, |\phi_1\rangle, \ldots, |\phi_{15}\rangle\}$ can be computed easily and can be found in appendix A. The above eigenvalues and eigenstates completely determine the thermal states. The density operator $\rho(T)$ at the temperature T can be written as

$$\rho(T) = \frac{1}{Z}\sum_{\mu=0}^{15} \mathrm{e}^{-\beta E_\mu} |\phi_\mu\rangle\langle\phi_\mu| \tag{4}$$

where $\beta = 1/T$ and the partition function

$$\begin{aligned}
Z &= \mathrm{Tr}(\mathrm{e}^{-\beta H}) = \sum_{\mu=0}^{15} \mathrm{e}^{-\beta E_\mu} \\
&= 4 + 4\cosh(3\beta) + 4\cosh\beta + 2\cosh(4\beta) + 2\cosh(2\beta).
\end{aligned} \tag{5}$$

3. Violation of 4-qubit Bell inequality and the threshold temperature

To test quantum nonlocality for the state $\rho(T)$, correlation function Q_{ijkl} should be computed. From the definition of Q_{ijkl} [8], we have

$$\begin{aligned}
Q_{ijkl} &= \mathrm{Tr}[\rho(\hat{n}_i \cdot \vec{\sigma}) \otimes (\hat{n}_j \cdot \vec{\sigma}) \otimes (\hat{n}_k \cdot \vec{\sigma}) \otimes (\hat{n}_l \cdot \vec{\sigma})] \\
&= \frac{1}{Z}\sum_{\mu=0}^{15} \mathrm{e}^{-\beta E_\mu}\, \mathrm{Tr}[|\phi_\mu\rangle\langle\phi_\mu|(\hat{n}_i \cdot \vec{\sigma}) \otimes (\hat{n}_j \cdot \vec{\sigma}) \otimes (\hat{n}_k \cdot \vec{\sigma}) \otimes (\hat{n}_l \cdot \vec{\sigma})] \\
&= \frac{1}{Z}\sum_{\mu=0}^{15} \mathrm{e}^{-\beta E_\mu} Q^\mu_{ijkl}
\end{aligned} \tag{6}$$

where $\hat{n}_\alpha = (\sin\theta_\alpha, 0, \cos\theta_\alpha), \alpha = i, j, k, l$. Q^μ_{ijkl} is the correlation function for the eigenstate $|\phi_\mu\rangle$,

$$Q^\mu_{ijkl} = \mathrm{Tr}[|\phi_\mu\rangle\langle\phi_\mu|(\hat{n}_i \cdot \vec{\sigma}) \otimes (\hat{n}_j \cdot \vec{\sigma}) \otimes (\hat{n}_k \cdot \vec{\sigma}) \otimes (\hat{n}_l \cdot \vec{\sigma})]. \tag{7}$$

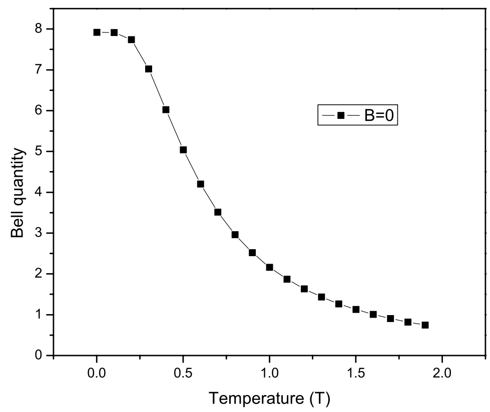

Figure 1. For a local realistic description of quantum mechanics, the Bell quantity $\mathcal{B}$ must necessarily be less than 4. However, the Bell quantity as a function of temperature T shows that there is a significant violation of the Bell inequality at $T < 0.626$.

For instance, the quantum correlation for the ground state $|\phi_5\rangle$ is given by

$$\begin{aligned} Q^5_{ijkl} &= \cos\theta_i\cos\theta_j\cos\theta_k\cos\theta_l + \frac{\sqrt{3}}{2}\cos\theta_k\cos\theta_l\sin\theta_i\sin\theta_j \\ &\quad - \frac{\sqrt{3}}{4}\cos\theta_j\cos\theta_l\sin\theta_i\sin\theta_k + \frac{1}{2}\cos\theta_i\cos\theta_l\sin\theta_j\sin\theta_k \\ &\quad + \frac{1}{2}\cos\theta_j\cos\theta_k\sin\theta_i\sin\theta_l - \frac{\sqrt{3}}{4}\cos\theta_i\cos\theta_k\sin\theta_j\sin\theta_l \\ &\quad + \frac{\sqrt{3}}{2}\cos\theta_i\cos\theta_j\sin\theta_k\sin\theta_l + \sin\theta_i\sin\theta_j\sin\theta_k\sin\theta_l. \end{aligned} \tag{8}$$

Other quantum correlation functions can also be calculated in a similar way. The correlation function for the thermal state $\rho(T)$ is computed using equation (6). Based on the calculated values of Q_{ijkl}, we construct Bell quantity $\mathcal{B}$

$$\begin{aligned} \mathcal{B} &= Q_{1111} - Q_{1112} - Q_{1121} - Q_{1122} - Q_{1211} - Q_{1212} - Q_{1221} + Q_{1222} \\ &\quad - Q_{2111} - Q_{2112} - Q_{2121} + Q_{2122} - Q_{2211} + Q_{2212} + Q_{2221} + Q_{2222}. \end{aligned} \tag{9}$$

For a local realistic description, we require $-4 \leqslant \mathcal{B} \leqslant 4$. In figure 1, we have numerically computed the Bell quantity as a function of temperature. The results show that violation of the Bell inequality occurs at $T \leqslant T_0 = 0.626$. We call this critical value T_0 the threshold temperature. The maximum value of $\mathcal{B}$ for the state $\rho(T)$ approaches 7.917 at temperature close to zero.

We have also evaluated the Bell quantity $\mathcal{B}(|\phi_\mu\rangle)$ in terms of correlation functions of each pure state $|\phi_\mu\rangle$. The maximum values of $\mathcal{B}(|\phi_\mu\rangle)$ are

$$\begin{aligned} \mathcal{B}_{\max}(|\phi_\mu\rangle) = 4 &\qquad \text{for } |\phi_{0,15}\rangle \\ 6.112 &\qquad \text{for } |\phi_{1,2,3,4,11,12,13,14}\rangle \\ 7.917 &\qquad \text{for } |\phi_{5,10}\rangle \\ 5.657 &\qquad \text{for } |\phi_{6,9}\rangle \\ 4.866 &\qquad \text{for } |\phi_7\rangle \\ 4.060 &\qquad \text{for } |\phi_8\rangle. \end{aligned} \tag{10}$$

We can explain qualitatively why the maximum value of $\mathcal{B}$ for the thermal state should be 7.917 by noting that the thermal state $\rho(T)$ is the linear combination of $|\phi_\mu\rangle\langle\phi_\mu|$ weighted with the factors $e^{-\beta E_\mu}$. For eigenvalue $E_5 = -4, \mathcal{B}_{\max}(|\phi_5\rangle) = 7.917$, the power is $e^{4\beta}$ and when β is large enough, the Bell quantity $\mathcal{B}$ is totally determined by the contribution of state $|\phi_5\rangle$. Another thing worth noting is that the eigenstates of extended XX model do not lead to the highest value of $\mathcal{B}_{\max}$. We check that the maximum value of the Bell quantities consists of correlation functions for the following three general states,

$$|\phi'\rangle = \cos\alpha_1|1000\rangle + \sin\alpha_1\cos\alpha_2|0100\rangle + \sin\alpha_1\sin\alpha_2\cos\alpha_3|0010\rangle \\ + \sin\alpha_1\sin\alpha_2\sin\alpha_3|0001\rangle \tag{11}$$

$$|\phi''\rangle = \cos\alpha_1|1110\rangle + \sin\alpha_1\cos\alpha_2|1101\rangle + \sin\alpha_1\sin\alpha_2\cos\alpha_3|1011\rangle \\ + \sin\alpha_1\sin\alpha_2\sin\alpha_3|0111\rangle \tag{12}$$

$$|\phi'''\rangle = \cos\alpha_1|1100\rangle + \sin\alpha_1\cos\alpha_2|1010\rangle + \sin\alpha_1\sin\alpha_2\cos\alpha_3|1001\rangle \\ + \sin\alpha_1\sin\alpha_2\sin\alpha_3\cos\alpha_4|0110\rangle + \sin\alpha_1\sin\alpha_2\sin\alpha_3\sin\alpha_4\cos\alpha_5|0101\rangle \\ + \sin\alpha_1\sin\alpha_2\sin\alpha_3\sin\alpha_4\sin\alpha_5|0011\rangle \tag{13}$$

and find that

$$\mathcal{B}_{\max}(|\phi_0'\rangle) = 6.217 \qquad \mathcal{B}_{\max}(|\phi_0''\rangle) = 6.217 \qquad \mathcal{B}_{\max}(|\phi_0'''\rangle) = 8.485 \tag{14}$$

for $|\phi_0'\rangle = 1/2(|1000\rangle + |0100\rangle + |0010\rangle + |0001\rangle)$, $|\phi_0''\rangle = 1/2(|1110\rangle + |1101\rangle + |1011\rangle + |0111\rangle)$ and $|\phi_0'''\rangle = 1/\sqrt{6}(|1100\rangle + |1010\rangle + |1001\rangle + |0110\rangle + |0101\rangle + |0011\rangle)$ respectively. It is easy to see that the degree of violation of Bell's inequality for state $|\phi_0'\rangle$ is higher than that for the eigenstates $|\phi_\mu\rangle(\mu = 1, 2, 3, 4)$ listed in equation (A.2). The same results also happen for the eigenstates $|\phi_\mu\rangle(\mu = 11, 12, 13, 14)$ and $|\phi_\mu\rangle(\mu = 5, 6, 7, 8, 9, 10)$ respectively. We see that among all possible $\mathcal{B}_{\max}$, the state $|\phi_0'''\rangle$ yields the largest violation.

4. The effect of an external magnetic field

In this section, we would like to study the effect of a magnetic field on the nonlocality property of a thermal state in a general way, for which the Hamiltonian becomes

$$H' = 2\sum_{n=1}^{N-1} J_{n,n+1}\left(\sigma_n^+\sigma_{n+1}^- + \sigma_n^-\sigma_{n+1}^+\right) + B\sum_{n=1}^{N}\sigma_z \tag{15}$$

where B is the strength of the magnetic field. It is easy to verify that the eigenstates of H' are identical with those listed in expression (A.2) of H, but with different eigenvalues

$$\begin{array}{llll} E_0' = 4B, & E_1' = -3 + 2B, & E_2' = 3 + 2B, & E_3' = -1 + 2B, \\ E_4' = 1 + 2B, & E_5' = -4, & E_6' = -2, & E_7' = 0, \\ E_8' = 0, & E_9' = 2, & E_{10}' = 4, & E_{11}' = -3 - 2B, \\ E_{12}' = 3 - 2B, & E_{13}' = -1 - 2B, & E_{14}' = 1 - 2B, & E_{15}' = -4B \end{array} \tag{16}$$

and hence, a new correlation function and Bell quantity $\mathcal{B}'$ are given

$$Q_{ijkl}' = \frac{1}{Z'}\sum_{\mu=0}^{15} e^{-\beta E_\mu'} Q_{ijkl}^\mu \tag{17}$$

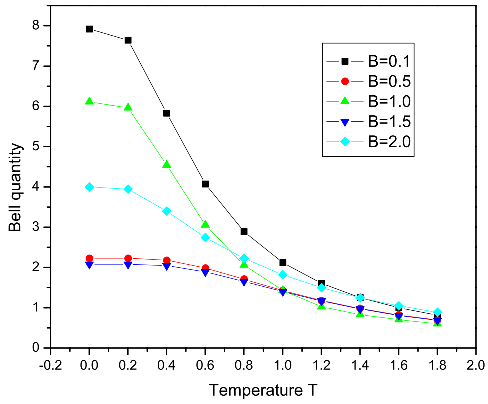

Figure 2. The Bell quantity for the cases with magnetic field $B = 0.1, 0.5, 1.0, 1.5$ and 2. (This figure is in colour only in the electronic version)

Table 1. Threshold temperatures for different strengths of the external magnetic field. When $B = 0.5$ and $B = 1.5$ and above, the values of the Bell quantity are not greater than 4 at all times. Therefore, no threshold temperatures exist for these cases.

B	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7
T_0	0.626	0.611	0.556	0.447	0.248	None	0.122	0.243
B	0.8	0.9	1.0	1.1	1.2	1.3	1.4	1.5 and above
T_0	0.351	0.427	0.467	0.472	0.436	0.343	0.18	None

$$\begin{aligned}\mathcal{B}' = Q'_{1111} - Q'_{1112} - Q'_{1121} - Q'_{1122} - Q'_{1211} - Q'_{1212} - Q'_{1221} + Q'_{1222} \\ - Q'_{2111} - Q'_{2112} - Q'_{2121} + Q'_{2122} - Q'_{2211} + Q'_{2212} + Q'_{2221} + Q'_{2222}\end{aligned} \tag{18}$$

where $Z' = \mathrm{Tr}(\mathrm{e}^{-\beta H'})$. Now the violation of the Bell inequality depends not only on the temperature, but also on the external magnetic field. Our numerical calculations are shown in figure 2.

There are five curves corresponding to $B = 0.1, 0.5, 1.0, 1.5$ and 2 respectively. When $B = 0.1$, the Bell quantity shows a similar variation of the violation of the Bell inequality as a function of T in the absence of a magnetic field. With increasing value of the external magnetic field, the maximum value of the Bell quantity approaches the value 2 for which the B field is about 1.5. The variation of the Bell quantity as a function of magnetic field can be explained qualitatively as follows. The $\rho'(T)$ is a different combination of $|\phi_\mu\rangle\langle\phi_\mu|$ compared with $\rho(T)$. The largest contribution of all the states $|\phi_\mu\rangle$ is determined by the value of B. When $B < 0.5$, it is the eigenstate, $|\phi_5\rangle$, which ultimately determines the maximal value of the Bell quantity ($\mathcal{B}_{\max} = 7.917$) since $\mathrm{e}^{-\beta E'_5} = \mathrm{e}^{4\beta}$ is the largest power among all the factors. When $0.5 < B < 1.5$, $|\phi_{11}\rangle$ takes the place of $|\phi_5\rangle$ with power $\mathrm{e}^{(3+2B)\beta}$ and $\mathcal{B}_{\max} = 6.112$ at $B = 1.0$, for example. When $B > 2$, $\mathrm{e}^{-\beta E'_{15}} = \mathrm{e}^{4B\beta}$ is the one with the largest contribution and $\mathcal{B}_{\max} = 4$. But there are two singular values of $B = 0.5$ and 1.5. In these two cases, $\mathcal{B}_{\max} < 4$. The reason for this is that the largest factors of $\mathrm{e}^{-\beta E'_\mu}$ are $\mathrm{e}^{-\beta E'_5} = \mathrm{e}^{-\beta E'_{11}} = \mathrm{e}^{4\beta}$ for $B = 0.5$, $\mathrm{e}^{-\beta E'_{15}} = \mathrm{e}^{-\beta E'_{11}} = \mathrm{e}^{6\beta}$ for $B = 1.5$, respectively. Thus the Bell quantity is determined principally using a combination of these two elements of Q^μ_{ijkl}, namely, $\mathrm{e}^{4\beta}\left(Q^5_{ijkl} + Q^{11}_{ijkl}\right)$ and $\mathrm{e}^{6\beta}\left(Q^{15}_{ijkl} + Q^{11}_{ijkl}\right)$. Note that the maximum values of the Bell quantity for the latter two correlation functions are 2.228 and 2.081 respectively.

The critical temperatures under different magnetic fields have been found (table 1). The variation of T_0 with increasing strength of B is more complicated. This complication arises mainly because the eigenstates contributing to the optimization of critical temperatures are different from those needed for the optimization of magnetic fields. In the latter case, $\mathcal{B}_{\max}$ is totally determined by the contribution of the state with the largest weight or factor for sufficiently large β. In the former case, depending on the value of the external magnetic field, the eigenstates contributing to the optimization changes and so the optimization are determined using a combination of the correlation functions from different states. In short, the variation of T_0 with B is different from that of $\mathcal{B}_{\max}$ with B.

5. Conclusion

In this paper, we consider the extended Heisenberg XX model, modelling the nearest-neighbour interaction spin chain. For the 4-qubit extended XX model, it is shown that since the correlation functions depend on the temperature and the magnetic field, the violation of the Bell inequality for the thermal state depends critically on these two parameters. The effect of temperature for a local realistic description of quantum theory is determined by the threshold value of T below which the thermal state violates the Bell inequality. The effects of temperature are also studied at different strengths of magnetic field. For a fixed temperature, we can find the optimal value of the external magnetic field for the violation of Bell inequalities. Our results imply that quantum 'nonlocality' could be effectively controlled by magnetic field and temperature. We restrict ourselves to the 4-qubit case. However, we could also have discussed the violation of the Bell inequality for the thermal state for the 2-qubit and 3-qubit cases. For the 2-qubit extended XX model, the Bell quantity approaches $2\sqrt{2}$ which is the maximal violation of the 2-qubit Bell inequality and the corresponding threshold value of temperature is $T_0 = 0.667$ when $B = 0$. However, for the 3-qubit case, the correlation function defined by this method is always equal to 0. The violation of the Bell inequality for arbitrary number of qubits can also be done in the same manner.

Acknowledgments

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Appendix A. Eigenstates of the 4-qubit Hamiltonian

Corresponding to the 16 eigenvalues of the Hamiltonian

$$\begin{aligned} &E_0 = E_7 = E_8 = E_{15} = 0, \\ &E_3 = E_{13} = -1, \qquad E_4 = E_{14} = 1, \\ &E_6 = -2, \qquad E_9 = 2, \\ &E_1 = E_{11} = -3, \qquad E_2 = E_{12} = 3, \\ &E_5 = -4, \qquad E_{10} = 4, \end{aligned} \tag{A.1}$$

the orthogonal eigenstates are

$$|\phi_0\rangle = |0000\rangle$$

$$|\phi_1\rangle = \tfrac{1}{2\sqrt{2}}(-|1000\rangle + \sqrt{3}|0100\rangle - \sqrt{3}|0010\rangle + |0001\rangle)$$

$$
\begin{aligned}
|\phi_2\rangle &= \tfrac{1}{2\sqrt{2}}(|1000\rangle + \sqrt{3}|0100\rangle + \sqrt{3}|0010\rangle + |0001\rangle) \\
|\phi_3\rangle &= \tfrac{\sqrt{3}}{2\sqrt{2}}\big(|1000\rangle - \tfrac{1}{\sqrt{3}}|0100\rangle - \tfrac{1}{\sqrt{3}}|0010\rangle + |0001\rangle\big) \\
|\phi_4\rangle &= \tfrac{\sqrt{3}}{2\sqrt{2}}\big(-|1000\rangle - \tfrac{1}{\sqrt{3}}|0100\rangle + \tfrac{1}{\sqrt{3}}|0010\rangle + |0001\rangle\big) \\
|\phi_5\rangle &= \tfrac{1}{4}(|1100\rangle - 2|1010\rangle + \sqrt{3}|1001\rangle + \sqrt{3}|0110\rangle - 2|0101\rangle + |0011\rangle) \\
|\phi_6\rangle &= \tfrac{1}{2}(-|1100\rangle + |1010\rangle - |0101\rangle + |0011\rangle) \\
|\phi_7\rangle &= \tfrac{\sqrt{3}}{\sqrt{10}}\big(|1100\rangle - \tfrac{2}{\sqrt{3}}|1001\rangle + |0011\rangle\big) \\
|\phi_8\rangle &= \tfrac{5}{2\sqrt{10}}\big(-\tfrac{\sqrt{3}}{5}|1100\rangle - \tfrac{3}{5}|1001\rangle + |0110\rangle - \tfrac{\sqrt{3}}{5}|0011\rangle\big) \\
|\phi_9\rangle &= \tfrac{1}{2}(-|1100\rangle - |1010\rangle + |0101\rangle + |0011\rangle) \\
|\phi_{10}\rangle &= \tfrac{1}{4}(|1100\rangle + 2|1010\rangle + \sqrt{3}|1001\rangle + \sqrt{3}|0110\rangle + 2|0101\rangle + |0011\rangle) \\
|\phi_{11}\rangle &= \tfrac{1}{2\sqrt{2}}(-|1110\rangle + \sqrt{3}|1101\rangle - \sqrt{3}|1011\rangle + |0111\rangle) \\
|\phi_{12}\rangle &= \tfrac{1}{2\sqrt{2}}(|1110\rangle + \sqrt{3}|1101\rangle + \sqrt{3}|1011\rangle + |0111\rangle) \\
|\phi_{13}\rangle &= \tfrac{\sqrt{3}}{2\sqrt{2}}\big(|1110\rangle - \tfrac{1}{\sqrt{3}}|1101\rangle - \tfrac{1}{\sqrt{3}}|1011\rangle + |0111\rangle\big) \\
|\phi_{14}\rangle &= \tfrac{\sqrt{3}}{2\sqrt{2}}\big(-|1110\rangle - \tfrac{1}{\sqrt{3}}|1101\rangle + \tfrac{1}{\sqrt{3}}|1011\rangle + |0111\rangle\big) \\
|\phi_{15}\rangle &= |1111\rangle.
\end{aligned} \tag{A.2}
$$

Appendix B. Quantum correlation functions for each pure state

The calculation of the quantum correlation functions is straightforward. In this appendix, we list all the correlation functions for each eigenstate of the 4-qubit Hamiltonian for easy reference.

Correlation function	Explicit expression
$Q^0_{ijkl} = Q^{15}_{ijkl}$	$\cos\theta_i \cos\theta_j \cos\theta_k \cos\theta_l$
$Q^1_{ijkl} = Q^{11}_{ijkl}$	$-\cos\theta_i \cos\theta_j \cos\theta_k \cos\theta_l - \frac{\sqrt{3}}{4}\cos\theta_k \cos\theta_l \sin\theta_i \sin\theta_j$
	$+\frac{\sqrt{3}}{4}\cos\theta_j \cos\theta_l \sin\theta_i \sin\theta_k - \frac{3}{4}\cos\theta_i \cos\theta_l \sin\theta_j \sin\theta_k$
	$-\frac{1}{4}\cos\theta_j \cos\theta_k \sin\theta_i \sin\theta_l + \frac{\sqrt{3}}{4}\cos\theta_i \cos\theta_k \sin\theta_j \sin\theta_l$
	$-\frac{\sqrt{3}}{4}\cos\theta_i \cos\theta_j \sin\theta_k \sin\theta_l$
$Q^2_{ijkl} = Q^{12}_{ijkl}$	$-\cos\theta_i \cos\theta_j \cos\theta_k \cos\theta_l + \frac{\sqrt{3}}{4}\cos\theta_k \cos\theta_l \sin\theta_i \sin\theta_j$
	$+\frac{\sqrt{3}}{4}\cos\theta_j \cos\theta_l \sin\theta_i \sin\theta_k + \frac{3}{4}\cos\theta_i \cos\theta_l \sin\theta_j \sin\theta_k$
	$+\frac{1}{4}\cos\theta_j \cos\theta_k \sin\theta_i \sin\theta_l + \frac{\sqrt{3}}{4}\cos\theta_i \cos\theta_k \sin\theta_j \sin\theta_l$
	$+\frac{\sqrt{3}}{4}\cos\theta_i \cos\theta_j \sin\theta_k \sin\theta_l$
$Q^3_{ijkl} = Q^{13}_{ijkl}$	$-\cos\theta_i \cos\theta_j \cos\theta_k \cos\theta_l - \frac{\sqrt{3}}{4}\cos\theta_k \cos\theta_l \sin\theta_i \sin\theta_j$
	$-\frac{\sqrt{3}}{4}\cos\theta_j \cos\theta_l \sin\theta_i \sin\theta_k + \frac{1}{4}\cos\theta_i \cos\theta_l \sin\theta_j \sin\theta_k$

	$+\frac{3}{4}\cos\theta_j\cos\theta_k\sin\theta_i\sin\theta_l - \frac{\sqrt{3}}{4}\cos\theta_i\cos\theta_k\sin\theta_j\sin\theta_l$
	$-\frac{\sqrt{3}}{4}\cos\theta_i\cos\theta_j\sin\theta_k\sin\theta_l$
$Q^4_{ijkl} = Q^{14}_{ijkl}$	$-\cos\theta_i\cos\theta_j\cos\theta_k\cos\theta_l + \frac{\sqrt{3}}{4}\cos\theta_k\cos\theta_l\sin\theta_i\sin\theta_j$
	$-\frac{\sqrt{3}}{4}\cos\theta_j\cos\theta_l\sin\theta_i\sin\theta_k - \frac{1}{4}\cos\theta_i\cos\theta_l\sin\theta_j\sin\theta_k$
	$-\frac{3}{4}\cos\theta_j\cos\theta_k\sin\theta_i\sin\theta_l - \frac{\sqrt{3}}{4}\cos\theta_i\cos\theta_k\sin\theta_j\sin\theta_l$
	$+\frac{\sqrt{3}}{4}\cos\theta_i\cos\theta_j\sin\theta_k\sin\theta_l$
Q^6_{ijkl}	$\cos\theta_i\cos\theta_j\cos\theta_k\cos\theta_l + \cos\theta_i\cos\theta_l\sin\theta_j\sin\theta_k$
	$-\cos\theta_j\cos\theta_k\sin\theta_i\sin\theta_l - \sin\theta_i\sin\theta_j\sin\theta_k\sin\theta_l$
Q^7_{ijkl}	$\cos\theta_i\cos\theta_j\cos\theta_k\cos\theta_l + \frac{2\sqrt{3}}{5}\cos\theta_j\cos\theta_l\sin\theta_i\sin\theta_k$
	$\frac{2\sqrt{3}}{5}\cos\theta_i\cos\theta_k\sin\theta_j\sin\theta_l + \frac{3}{5}\sin\theta_i\sin\theta_j\sin\theta_k\sin\theta_l$
Q^8_{ijkl}	$\cos\theta_i\cos\theta_j\cos\theta_k\cos\theta_l + \frac{\sqrt{3}}{10}\cos\theta_j\cos\theta_l\sin\theta_i\sin\theta_k$
	$\frac{\sqrt{3}}{10}\cos\theta_i\cos\theta_k\sin\theta_j\sin\theta_l - \frac{3}{5}\sin\theta_i\sin\theta_j\sin\theta_k\sin\theta_l$
Q^9_{ijkl}	$\cos\theta_i\cos\theta_j\cos\theta_k\cos\theta_l - \cos\theta_i\cos\theta_l\sin\theta_j\sin\theta_k$
	$+\cos\theta_j\cos\theta_k\sin\theta_i\sin\theta_l - \sin\theta_i\sin\theta_j\sin\theta_k\sin\theta_l$
Q^{10}_{ijkl}	$\cos\theta_i\cos\theta_j\cos\theta_k\cos\theta_l - \frac{\sqrt{3}}{2}\cos\theta_k\cos\theta_l\sin\theta_i\sin\theta_j$
	$-\frac{\sqrt{3}}{4}\cos\theta_j\cos\theta_l\sin\theta_i\sin\theta_k - \frac{1}{2}\cos\theta_i\cos\theta_l\sin\theta_j\sin\theta_k$
	$-\frac{1}{2}\cos\theta_j\cos\theta_k\sin\theta_i\sin\theta_l - \frac{\sqrt{3}}{4}\cos\theta_i\cos\theta_k\sin\theta_j\sin\theta_l$
	$-\frac{\sqrt{3}}{2}\cos\theta_i\cos\theta_j\sin\theta_k\sin\theta_l + \sin\theta_i\sin\theta_j\sin\theta_k\sin\theta_l$

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